Title: Identification of Damping Properties of Viscoelastic Layers

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ABSTRACT

New methodology of evaluating damping properties of middle viscoelastic layer in three layered beams is presented. When the composite structure with two outer elastic layers is subjected to harmonic transverse load, the specific mechanism of damping is triggered. Flexure of the structure produces extensional strains in the face layers while shear strains prevail in the middle layer. This way of dissipation of energy is widely known as constrained layer damping. It is distinct from free layer damping where extensional strains are induced in viscoelastic layer.

Mathematical formulation of the issue leads to an inverse problem. MSC. NASTRAN Finite Element Method program is used to analyse response of the structure tested in simultaneously performed experiment when different values of viscoelastic parameters are taken. The values are changed in the consecutive iterations. In every step the numerical response of the system subjected to harmonic excitation is compared with experimental measurements. This procedure is repeated as long as the best fit is reached. Cantilever beam excited at the free end by electromagnetic transducer is used in experiment. An inductive transducer measures displacements.

Application of the algorithm for other structures is discussed. Alternative solution is proposed.

INTRODUCTION

Materials of viscoelastic properties exhibit internal damping. The advantage of it is taken to supress vibrations of low-stiffness structures subjected to periodic excitation. Viscoelastic tapes and films are applied to car body, aircraft

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skin and other elements to avoid resonance. Mechanical description has been employed to survey behaviour of such multilayer systems. In most cases they can be approximated by models of plates and beams. The paper is devoted to the latter ones.

Amount of the energy dissipated by such structures depends to a high degree on the mechanism of deformation that is induced in a viscoelastic material. Thus arrangement of layers plays an important role. One of the most common configurations comprises viscoelastic core covered with usually metallic sheet and bonded to vibrating layer. When layers are set in this manner shear prevail in the core while face sheets undergo extension. Mechanism of dissipation of energy observed in this system is called constrained layer damping.

Differential equation governing vibrations of three-layer simply supported beam was derived for the first time by Kerwin [1]. Di Taranto [2] founded his analysis on assumptions made by Kerwin except he permitted arbitrary boundary conditions. He obtained sixth-order differential equations. Thorough discussion of this solution and extended examination of the case of clamped-clamped sandwich beam was given by Mead and Markus in [3] and [4], respectively. Yan and Dowell [5] deduced exact and simplified equations (of forth order) on the basis of analysis of state of stress and strain in individual layers and application of interface conditions. Energy approach grounded on Hamilton's principle was presented by Rao [6].

Numerous treatises [7], [8], [9] are also devoted to numerical formulation of the problem. Various finite element method algorithms are proposed to model properly behaviour of multilayer damped structures and compute their dynamic response.

All the aforementioned works focus either on determination of some parameters characterizing homogenized structure, solution of the problem of free and forced vibrations or optimal design of the composite. However the need sometimes arises to find out material parameters of viscoelastic core when dynamic response of the system is known. Altough there exist many well-established methods for direct measurement of such quantities [10] solution of inverted problem still seems to be advantageous since the process of joining layers affects material properties.

In this paper the algorithm enabling one to determine viscoelastic properties of middle layer in three-layer cantilever beams on the basis of frequency response of the structure is presented. The latter data is obtained from experiment. Numerical simulation of the dynamic tests is performed and model updating procedure is developed where values of parameters to be found are altered. MSC.Nastran programme is used for Finite Element Method computations. Outer layers are assumed to be made up of elastic materials of known parameters.

FINITE ELEMENT METHOD MODEL

Three dimensional model is constructed. Every sheet of a sandwich beam is modelled by one layer of isoparametric 8-node CHEXA solid elements (see Figure 1). Isotropic material description is applied for the whole structure. Face layers are assumed to be elastic. The viscoelastic core is represented by three parameters. Poisson ratio value is taken to be 0.40. The other two quantities that

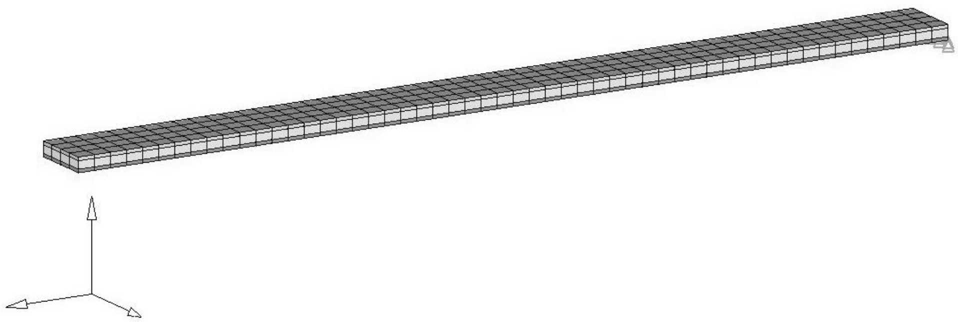

Figure 1. Finite element model of three-layer beam.

are the unknown to be determined relate state of stress and strain in following manner:

$$\underline{\sigma}(\omega) = E(\omega)\left[1 + in(\omega)\right]\underline{\varepsilon}(\omega) \quad (1)$$

where i is imaginary unit, $i^2 = -1$, $E(\omega)$ is storage modulus and $\eta(\omega)$ is loss factor.

NUMERICAL ANALYSIS

The procedure for independent determination of the two unknown parameters is discussed below. The presented algorithm has been implemented in MATLAB programme. It uses MSC.Nastran solver to perform Finite Element Method analysis.

Storage Modulus Determination

Modal analysis of the structure is carried out for different values of storage modulus until required agreement of numerical results with experimental response is reached. Since storage modulus depends on frequency this iterative process has to be repeated for every resonant frequency observed in experiment. To speed up the search of correct values the optimisation algotithm based on Nelder-Mead simplex algorithm [11] is implemented. Finally set of modal values of the parameter is obtained. In every iteration real eigenvalue problem is solved

$$\left[\mathbf{K} - \omega^2\mathbf{M}\right]\{\Phi\} = \{0\} \quad (2)$$

Influence of loss factor is neglected here. To get exact values of eigenfrequencies complex eigenvalue analysis should be performed. Loss factor would enter equation (2) but independent determination of the parameters would be impossible.

Loss Factor Determination

Process of determination of loss factor is analogous to that described in previous chapter. Herein frequency domain analysis is performed and shape of resonant peaks (that is proved to be dependent on loss factor [12]) obtained from experiment and numerical calculations is compared. Values of storage modulus determined in previous procedure are used. Equation of motion for harmonic excitation has the form

$$\left[-\omega^2\mathbf{M}+\mathbf{K}(1+i\eta)\right]\{\mathbf{u}(\omega)\}=\{\mathbf{F}(\omega)\} \tag{3}$$

Complex stiffness matrix is obtained because hysteretic model of material damping has been used. Constitutive equation (1) is still valid but it is proved [13] that unlike when viscous damping model is applied if set of equations (3) is uncoupled loss factor does not affect value of resonant frequency and hence storage modulus stays unaltered. Unfortunatelly this property cannot be fully exploited here since in general damped modes do not satisfy orthogonality conditions and thus do not allow uncoupled equations of motion (3) to be obtained. Optimisation procedure based on golden section search and parabolic interpolation [14] is implemented to seek loss factor values.

CONCLUSIONS

Numerical procedure for determination of properties of viscoelastic layer in sandwich beams has been presented. An example of three-layer beam has been investigated. It should be emphasized that the same algorithm can be used for other structures with arbitrary arrangement of layers provided that required experimental data is available. Application of Finite Element Method enables one to reject kinematical constraints imposed in theoretical formulation of the problem but as it has been shown above some discrepancies are encountered.

Time domain formulation is planned to be developed in near future. This requires modification of experimental stand (faster excitation transducer and pick-up sensor should be mounted to allow generation and detection of impact signal) but it is hoped to yield more data needed to determine the unknown parameters.

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